

# Warm up

Find a numerical value

1) If  $\frac{\cot x}{\cos x} = 2$  find the value of  $\sin x$ .

$$\frac{\frac{\cos x}{\sin x}}{\cos x}$$
$$\frac{\cos x}{\sin x} \cdot \frac{1}{\cos x}$$

means

$$\sin x =$$

Simplify left side first

$$\frac{1}{\sin x}$$
$$\csc x = 2$$

$$\sin x = \frac{1}{2}$$

2) If  $\cos^2 x + 2\sin x - 2 = 0$  find the value of  $\sin x$ .

$$\sin x =$$

$$1 - \sin^2 x + 2\sin x - 2 = 0$$

$$-\sin^2 x + 2\sin x - 1 = 0$$

$$\sin^2 x - 2\sin x + 1 = 0$$

$$(\sin x - 1)^2 = 0$$

$$\sin x = 1$$

3) If  $\sin x \cot x = 1$  find the value of  $\cos x$ .

$$\cos x =$$

$$\sin x \cdot \frac{\cos x}{\sin x} = 1$$

$$\cos x = 1$$

4) If  $\sin x \sec x = 3$  find the value of  $\cot x$ .

$$\cot x =$$

$$\sin x \cdot \frac{1}{\cos x} = 3$$

$$\frac{\sin x}{\cos x}$$

$$\tan x = 3$$

$$\cot x = \frac{1}{3}$$

## 5.4 Notes: Double Angle Formulas

### Double Angle Identities

$$\sin 2u = 2 \sin u \cos u$$

$$\cos 2u = \cos^2 u - \sin^2 u$$

$$\tan 2u = \frac{2 \tan u}{1 - \tan^2 u}$$

$$= 1 - 2 \sin^2 u$$

$$= 2 \cos^2 u - 1$$

**NOTE:**  $\sin^2 u \neq \sin 2u$

$\cos^2 u \neq \cos 2u$

$\tan^2 u \neq \tan 2u$

**Ex: 1** Solve

a.)  $2 \cos x + \sin 2x = 0$

b.)  $3 \sin^2 x + \cos 2x - 2 = 0$

a)  $2 \cos x + 2 \sin x \cos x = 0$

$$2 \cos x (1 + \sin x) = 0$$

$$\cos x = 0$$

$$\sin x = -1$$

$$\frac{\pi}{2} + 2\pi n$$

$$\frac{3\pi}{2} + 2\pi n$$

$$\frac{3\pi}{2} + 2\pi n$$

b)  $3 \sin^2 x + 1 - 2 \sin^2 x - 2 = 0$

$$\sin^2 x - 1 = 0$$

$$\sin x = \pm 1$$

$$\frac{\pi}{2} + 2\pi n$$

$$\frac{3\pi}{2} + 2\pi n$$

**You Try:** Solve  $\cos 2x + \cos x = 0$

$$2 \cos^2 x - 1 + \cos x = 0$$

$$2 \cos^2 x + \cos x - 1 = 0$$

$$(2 \cos x - 1)(\cos x + 1) = 0$$

$$\cos x = \frac{1}{2}$$

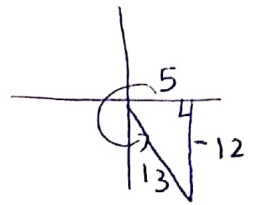
$$\cos x = -1$$

$$\frac{\pi}{3} + 2\pi n$$

$$\pi + 2\pi n$$

$$\frac{5\pi}{3} + 2\pi n$$

QIV  
 Ex: 2 Given  $\cos \theta = \frac{5}{13}$ , and  $\frac{3\pi}{2} < \theta < 2\pi$  find  $\sin 2\theta$ ,  $\cos 2\theta$ ,  $\tan 2\theta$ .

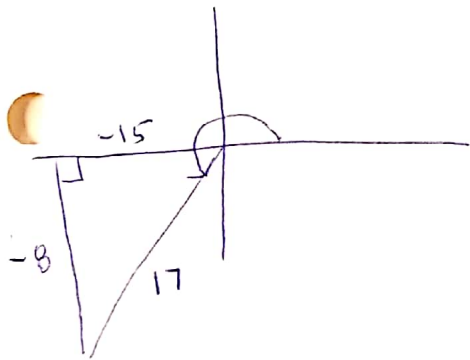


$$\begin{aligned} \sin 2\theta &= 2 \sin \theta \cos \theta \\ &= 2 \left( \frac{-12}{13} \right) \left( \frac{5}{13} \right) \\ &= \boxed{\frac{-120}{169}} \end{aligned}$$

$$\begin{aligned} \cos 2\theta &= \cos^2 \theta - \sin^2 \theta \\ &= \left( \frac{5}{13} \right)^2 - \left( \frac{-12}{13} \right)^2 \\ &= \frac{25}{169} - \frac{144}{169} \\ &= \boxed{\frac{-119}{169}} \end{aligned}$$

$$\begin{aligned} \tan 2\theta &= \frac{2 \tan \theta}{1 - \tan^2 \theta} \\ &= \frac{2 \left( \frac{-12}{5} \right)}{1 - \left( \frac{-12}{5} \right)^2} \\ &= \frac{-\frac{24}{5}}{\frac{25}{25} - \frac{144}{25}} = \frac{-\frac{24}{5}}{\frac{-119}{25}} \\ &= \frac{-24}{5} \cdot \frac{25}{-119} = \boxed{\frac{120}{119}} \end{aligned}$$

QIII  
 You Try: Given  $\tan u = \frac{8}{15}$ , and  $\pi < u < \frac{3\pi}{2}$ , find  $\sin 2\theta$ ,  $\cos 2\theta$ ,  $\tan 2\theta$ .



$$\begin{aligned} \sin 2\theta &= 2 \sin \theta \cos \theta \\ &= 2 \left( \frac{-8}{17} \right) \left( \frac{-15}{17} \right) \\ &= \boxed{\frac{240}{289}} \end{aligned}$$

$$\begin{aligned} \cos 2\theta &= \cos^2 \theta - \sin^2 \theta \\ &= \left( \frac{-15}{17} \right)^2 - \left( \frac{-8}{17} \right)^2 \\ &= \boxed{\frac{161}{289}} \end{aligned}$$

$$\begin{aligned} \tan 2\theta &= \frac{2 \tan \theta}{1 - \tan^2 \theta} \\ &= \frac{2 \left( \frac{8}{15} \right)}{1 - \left( \frac{8}{15} \right)^2} \\ &= \boxed{\frac{240}{161}} \end{aligned}$$